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DISINTEGRATION OF AN ARBITRARY DISCONTINUITY OF THE ELECTROMAGNETIC FIELD IN FERROMAGNETICS

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G.L. SEDOVA
(Moscow)
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The disintegration of an arbitrary discontinuity of electromagnetic parameters in nonconducting ferromagnetics is considered. Cases in which the dependence between magnetic permeability of the medium and the modulus of the external magnetic field intensity is linear(conforms to the Rayleigh law) and inversely proportional(satifies the Fröhlich-Kennelly formula) are investigated.

The problem of disintegration of an arbitrary discontinuity for a perfect gas was solved in [1], for media with an arbitrary equation of state and for combustible mixtures it was solved in [2] and [3], respectively, and in magnetohydrodynamics it was solved in [4].

1. Let the parameters of the electromagnetic field in a ferromagnetic be discontinuous in the plane $x = 0$ at the initial instant of time, and let the arbitrary discontinuity disintegrate at subsequent instants of time. Owing to the self-similarity of the problem, the discontinuity disintegrates into some combination of strong discontinuities and two types of electromagnetic waves [5]. Below, by analogy to magnetohydrodynamics, the light waves A propagating at the speed $v_A = c / \sqrt{\mu\varepsilon}$ (μ is the magnetic permeability and ε is the permittivity of the ferromagnetic) will be called Alfven waves or rotational discontinuities.

We shall first consider the disintegration of discontinuity in the case in which μ is the reciprocal of the external magnetic field intensity H $(1 / \mu = a + bH)$. The propagation velocities of single waves, strong discontinuities, and Alfven waves will be shown below to be such that two waves originating at the contact discontinuity K can propagate in opposite directions with the rotational discontinuity A following either the single wave R or the strong discontinuity S (Fig. 1).

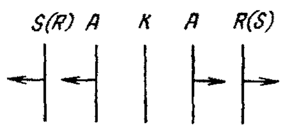


Fig. 1

Parameters of the medium to the left of the contact discontinuity plane are denoted below by a prime, whole those at the initial instant of time are denoted by subscript 0. Parameters of the medium behind the first wave propagating to the right or left bear subscript 1, and those of the medium behind discontinuity A bear subscript 2.

Let us investigate the relation between parameters in waves and at discontinuities in ferromagnetics when the magnetic permeability is inversely proportional to the modulus of the magnetic field intensity. It was shown in [6] that, if the normal components of the magnetic field at the discontinuity are initially unequal, a jump H_n occurs at the contact discontinuity and then the equality $[\mu H_n] = 0$ is satisfied. The propagation velocity of the contact discontinuity is zero, hence it lies at all times in the plane $x = 0$. Since the permittivity of the ferromagnetic ε is assumed here constant, hence the electric field normal component which satisfies the known relationships based on Maxwell equations cannot have discontinuities, and in that case the equality $[E_n] = 0$ is satisfied at all discontinuities and in waves.

Conditions at strong discontinuities are of the form

$$[E_z] = -\frac{c}{v_n\varepsilon}[H_y], \quad [E_y] = \frac{c}{v_n\varepsilon}[H_z] \tag{1.1}$$

$$v_n = \pm\frac{c}{\sqrt{\mu_1\varepsilon}}\left[1 - \frac{H_{2y}(\mu_1 - \mu_2)}{\mu_1 H_{1y} - \mu_2 H_{2y}}\right]^{1/2}$$

where v_n is the propagation velocity of the discontinuity, and the plus and minus signs correspond to waves that propagate, respectively, to the right and left.

The condition of evolution implies that the passage of a strong discontinuity increases the magnetic field intensity [6].

For a single wave the relationship between the electric and magnetic fields is of the form

$$E_z = E_{0z} - f(H;\ H_0), \quad E_y = E_{0y} + \varphi(H;\ H_0) \tag{1.2}$$

$$f(H; H_0) = \int_{H_0}^{H}\frac{c}{\lambda\varepsilon}\,dH_y, \quad \varphi(H; H_0) = \int_{H_0}^{H}\frac{c}{\lambda\varepsilon}\,dH_z$$

$$\lambda = \pm\frac{c}{\sqrt{\mu\varepsilon}}\left[1 + \frac{b\mu(H_y^2 + H_z^2)}{aH}\right]^{1/2}$$

where λ is the propagation velocity of single waves. It was shown in [6] that the derivative $d\lambda / dH > 0$, hence waves in which the magnetic field increases break, in fact, only waves is which the magnetic field decreases are realized.

A rotational discontinuity lags behind single and shock waves, since its propagation velocity is lower than that of the R-and S-waves. Such discontinuity turns vector H by some angle without affecting the magnitude of its modulus, with the relation between vectors $\mathbf{H}$ and $\mathbf{E}$ defined by formula $[\mathbf{E}] = \pm\sqrt{\mu / \varepsilon}\,[\mathbf{n} \times [\mathbf{H}]]$, where $\mathbf{n}$ is a vector normal to the discontinuity plane.

T h e c a s e o f p l a n e d i s c o n t i n u i t y. Let us, first, consider the plane problem of disintegration of an arbitrary discontinuity, in which vectors $\mathbf{H}_0 \perp \mathbf{E}_0$ $\mathbf{H}_0' \perp \mathbf{E}_0'$ and $\mathbf{H}_0 \parallel \mathbf{H}_0'$. If we select the coordinate system so that vector $\mathbf{H}_{0\tau}$ is directed along the y-axis, vectors $\mathbf{E}_{0\tau}$ and $\mathbf{E}_{0\tau}'$ will be directed along the z-axis. From the point with coordinates H_{0y} and E_{0z} (Fig. 2) we draw curves that correspond to oncoming and outgoing R- and S-waves whose equations are of the form

$$KR\colon\ E_z = E_{0z} - f(H_y; H_{0y}), \quad E_z > E_{0z} \tag{1.3}$$

$$KS\colon\ E_z = E_{0z} - \frac{c}{v_n\varepsilon}(H_y - H_{0y}), \quad E_z < E_{0z}$$

$$RK\colon\ E_z = E_{0z} + f(H_y; H_{0y}), \quad E_z > E_{0z}$$

$$SK\colon E_z = E_{0z} + \frac{c}{v_n\varepsilon}(H_y - H_{0y}), \quad E_z < E_{0z}$$

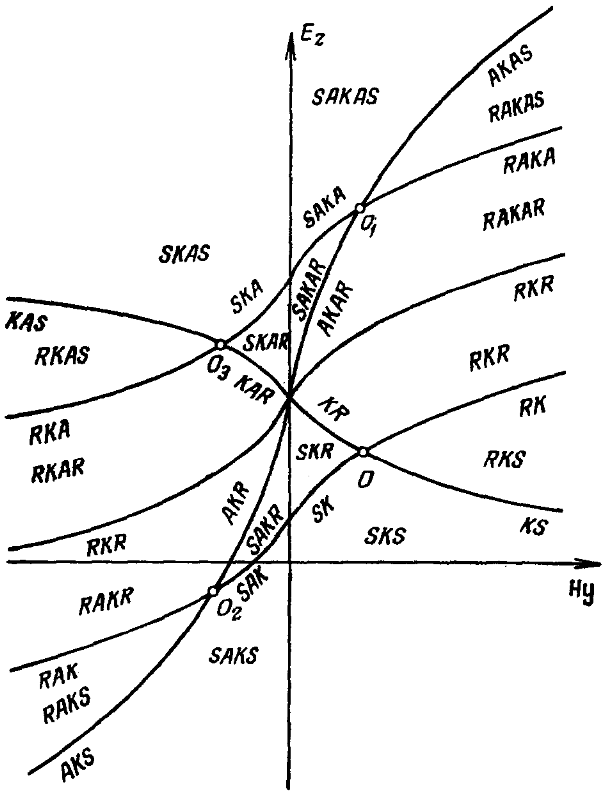


Fig. 2

The plane H_yE_z is divided near point O in four subregions in which one of the following four combinations of waves are possible: SKS, SKR, RKS and RKR.

When point (H_{0y}', E_{0z}') lies above line RKR issuing from the point of curve KR at $H = 0$, we have two Alfven discontinuities. When point H_{0y}', E_{0z}' lies on curve RKR, we have a combination of two single waves behind which the magnetic field dwindles to zero. The equation of the RKR -curve is of the form

$$E_{0z} = -f\,(0;\ H_{0y}) = E_z + f\,(0;\ H_y) \tag{1.4}$$

Let us consider point O_1 with coordinates $H = H_0$ and $E_{Az} = E_{0z} + 4H_0 \sqrt{\mu_0 / \varepsilon}$ which corresponds to the disintegration of the arbitrary discontinuity in two

A -discontinuities. From that point we draw curves $AKAR$, $RAKA$, $AKAS$, and $SAKA$. The equations of lines $RAKA$ and $SAKA$ are similar to Eqs. (1.3) for RK and SK, if we substitute in these E_{Az} for E_{0z}. The equations of lines $AKAR$ and $AKAS$ are of the form

$$E_z = E_{0z} - f(H_y H_{0y}) + 4H\sqrt{\mu/\varepsilon}, \quad 0 < H_y < H_{0y}$$

$$E_z = E_{0z} - \frac{c}{v_n \varepsilon}(H_y - H_{0y}) + 4H\sqrt{\mu/\varepsilon}, \quad H_y > H_{0y}$$

The whole right-hand half-plane $H_y > 0$ is thus divided in eight regions in each of which we have indicated the combination of waves that propagates there at the disintegration of the arbitrary discontinuity.

If $H_{0y}' < 0$, one Alfven discontinuity which turns the magnetic field by 180° must necessarily appear at disintegration, except when the magnetic field dwindles to zero along line RKR which may be continued beyond the axis $H_y = 0$. To the right and left of that line Alfven discontinuities are generated which propagate to the left and right, respectively. Let us consider points O_2 and O_3 which correspond to the passage of a single A-discontinuity. The coordinates of these points are, respectivley $H = -H_0$; $E_z = E_{0z} - 2H_0\sqrt{\mu_0/\varepsilon}$ and $H = -H_0$, $E_z = E_{0z} + 2H_0\sqrt{\mu_0/\varepsilon}$. The equations of lines AKS, RAK, SAK, AKR and KAS, and RKA, SKA, and KAR that issue from points O_2 and O_3 can be written by analogy to the equations that define disintegrations $AKAR$, $AKAS$, $SAKA$ and $RAKA$, substituting the coordinates of point O_2 or O_3 for the coordinates of point O_1 and intensity $-H$ for H, and taking into account that only one Alfven discontinuity takes place.

The left-hand half-plane is thus divided in a number of regions each of which corresponds to a combination of waves consisting of a single A -discontinuity and two or one wave.

T h e t h r e e d i m e n s i o n a l d i s i n t e g r a t i o n. Let vectors $\mathbf{H}_{0\tau}$, $\mathbf{H}_{0\tau}'$ and $\mathbf{E}_{0\tau}$, $\mathbf{E}_{0\tau}'$ be nonparallel. Turning the coordinate system so that $H_{0z} = 0$, we construct the solution in the plane E_yE_z. Then in that plane certain curves correspond to the combination of two Alfven waves and one single or shock wave, while the regions comprised between these discontinuities correspond to the combination of two discontinuities and two waves.

The equation of curve $AKAR$ is of the form

$$\begin{aligned}
&E_{1y} = E_{0y}, \quad E_{1z} = E_{0z} - f(H_1 H_0) \\
&E_{2y} - E_{1y} = \sqrt{\mu_1/\varepsilon}\, H_{2z}, \quad E_{2z} - E_{1z} = -\sqrt{\mu_1/\varepsilon}\,(H_{2y} - H_{1y}) \\
&E_{0y}' - E_{2y} = -\sqrt{\mu_1/\varepsilon}\,(H_{0z}' - H_{2z}), \\
&E_{0z} - E_{2z} = \sqrt{\mu_1/\varepsilon}\,(H_{0y}' - H_{2y}) \\
&|H_2| = |H_0|, \quad H_{1y} = H_{0y}|H_0'|/|H_0|
\end{aligned} \tag{1.5}$$

For H_{0y} and H_{0y}' specified behind the discontinuity we have a system of eight equations for the determination of nine quantities H_{1y}, H_{2y}, H_{2z}, E_{1y}, E_{1z}, E_{2y}, E_{2z}, E_{0y}', and E_{0z}'.

The relationship of E_{0y}' and E_{0z}' that corresponds to the disintegration of discontinuity to the $AKAR$ combination can be expressed by the formula of the form

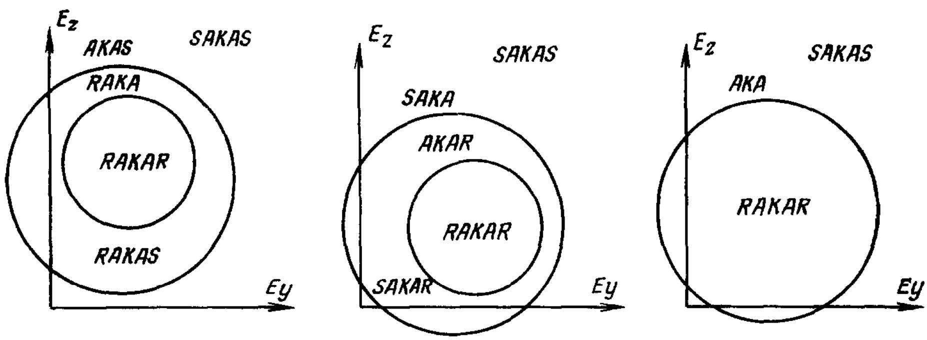
$$\left[E_{0y}' - E_{0y} - \sqrt{\frac{\mu_1}{\varepsilon}}\, H_{0z}'\right]^2 + \left[E_{0z}' - E_{0z} + f(H_1 H_0) - \sqrt{\frac{\mu_1}{\varepsilon}}\left(H_{0y}' + \frac{H_{y_0}|H_0'|}{|H_0|}\right)\right]^2 = 4\frac{\mu_1}{\varepsilon}\, H_0'^2 \qquad (1.6)$$

which represents a circle of radius $2\sqrt{\mu_1 / \varepsilon}\,|H_0'|$ whose center is at point

$$E_y = E_{0y} + \sqrt{\frac{\mu_1}{\varepsilon}}\, H_{0z}', \quad E_z = E_{0z} + \sqrt{\frac{\mu_1}{\varepsilon}} \times \left(H_{0y}' + \frac{H_{0y}|H_0'|}{|H_0|}\right)$$

It can be shown that the $AKAS$ line in the plane E_yE_z is also a circle.

When $|\mathbf{H}_{0\tau}| > |\mathbf{H}_{0\tau}'|$ we have in the plane E_yE_z two circles that correspond to the disintegration of discontinuity in a combination of $AKAS$ and $RAKA$ waves. Circle $RAKA$ lies inside circle $AKAS$ (Fig. 3, a), because in the plane case of disintegration of the arbitrary discontinuity the straight line $E_y = 0$) running from infinity, first, intersects the region to which corresponds combination $SAKAS$ and, then curve $AKAS$ and continues in region $RAKAS$. If $H_{0z}' = 0$, the $AKAS$ circle is symmetric about the axis $E_y = E_{0y}$; points E_{z1} and E_{z2} of intersection of circle $AKAS$ with the E_z -axis can be found in Fig. 2 for the plane disintegration of dis-



a b c

Fig. 3

continuity. These points correspond to the disintegration in a combination of KS and $AKAS$. By rotating these points about the middle of the segment that joins them, we can obtain the sought circle $AKAS$. Introduction of H_{0z}' results in the shift of the circle along the E_y -axis by $\sqrt{\mu_1 / \varepsilon} H_{0z}'$ and in the increase of its radius to $2\sqrt{\mu_1 \varepsilon}\,|H_0'|$.

When $|\mathbf{H}_{0\tau}'| < |\mathbf{H}_{0\tau}|$ we have in region E_yE_z also two circles, viz. $SAKA$ and $AKAR$ (the latter lying inside the first, Fig. 3, b), and when $|\mathbf{H}_{0\tau}| = |\mathbf{H}_{0\tau}'|$ the disintegration is only into the combinations $SAKAS$, $RAKAS$ and AKA (Fig. 3, c).

When field parameters are known on both sides of the discontinuity plane, then by constructing the curves defined by equations of the type (1.5), we obtain in the plane E_yE_z circles that are similar to those shown in Fig. 3. When the coordinates of point $(H_{0y}' \, H_{0z}')$ and $(E_{0y}' \, E_{0z}')$ are known, it is possible to determine the combination of waves that propagate for given parameter values.

2. Let us consider the disintegration of the magnetic field when the dependence of μ on H $(\mu = \mu_0 + \mu_H H)$ is linear. In that case the propagation velocity of single waves and strong discontinuities is lower than that of Alfven waves. A qualitative picture of the disintegration of arbitrary discontinutity of the magnetic field when the dependence of magnetic permeability on the magnetic field is linear is shown in Fig. 4. It was shown in [6] that in this case the normal component of the magnetic field H_n does not break at the contact discontinuity. The contact discontinuity virtually means only the plane $x = 0$.

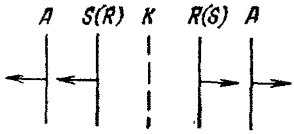


Fig. 4

The conditions at a strong discontinuity are of the same form as in the case of inverse dependence of μ on H (formula (1.1)). Strong discontinuities propagate at the velocity

$$v_n = \pm \frac{c}{\sqrt{\mu_1 \varepsilon}} \left[1 - \frac{\mu_H H_{2y} (H_1 - H_2)}{\mu_0 (H_{1y} - H_{2y}) + \mu_H (H_{1y} H_2 - H_{2y} H_1)} \right]^{1/2}$$

The condition of evolution implies that in a strong discontinuity the magnetic field decreases.

The relation between the electric and magnetic fields in single waves is of the same form as in formulas (1.2), and the propagation velocity of these waves is

$$\lambda = \pm \frac{c}{\sqrt{\mu \varepsilon}} \left[1 + \frac{\mu_H H_y^2}{(\mu + \mu_H H) H} \right]^{1/2}$$

Since the derivative $d\lambda / dH < 0$, hence waves in which the magnetic field decreases break and are converted to strong discontinuities. Only those single waves in which the magnetic field increase can exist.

When the dependence of μ on H is linear the rotational discontinuity is exactly the same as the one considered in Sectn. 1. In such discontinuity the modulus of vector H remains unchanged, hence the magnetic permeability and the propagation velocity of discontinuity are also constant.

T h e p l a n e c a s e o f d i s i n t e g r a t i o n. When the field parameters behind and ahead the discontinuity are related by expressions $\mathbf{H}_0 \perp \mathbf{E}_0, \mathbf{H}_0' \perp \mathbf{E}_0'$ and $\mathbf{H}_0 \parallel \mathbf{H}_0'$ the pattern of disintegration of an arbitrary discontinuity corresponds to that shown in Fig. 2. The problem of disintegration is solved similarly to that considered in Sectn. 1. The equations of lines KR, KS, RK and SK are of the form (1.3) in which the inequalities are reversed. The equation of line SKS is defined by formula

$$E_{0z} = \frac{c}{v_n \varepsilon} H_{0y} = E_z + \frac{c}{v_n \varepsilon} H_y$$

The equations of all remaining lines which correspond to the disintegration of a plane discontinuity in Alfven discontinuities and a single or shock wave are the same as in Sectn. 1, if the relationships for the R- and S-waves are substituted one for the other. It is necessary to take in that case into account that the Alfven wave overtakes the single wave and the strong discontinuity.

The three-dimensional case. When behind and ahead of a discontinuity vectors $\mathbf{H}_\tau$ and $\mathbf{E}_\tau$ are not parallel, the disintegration of such discontinuity is accompanied by the generation of rotational discontinuities that induce transverse components of the magnetic field. The equations that define the relation of components E_y and E_z in wave combinations consisting of two Alfven waves and one R- or S- wave coincide with formula (1.6). The plane E_yE_z is divided by circles $AKRA, AKSA, ASKA$ and $ARKA$ in regions in which combinations of waves $ARKRA$, $ARKSA$, $ASKRA$ and $ASKSA$ are realized.

By substituting in Fig. 3 the symbols S for R and R for S we obtain the pattern of three-dimensional disintegration when the dependence of on H is linear and $|\mathbf{H}_{0\tau}| > |\mathbf{H}_{0\tau}'|$ (a), and $|\mathbf{H}_{0\tau}| < |\mathbf{H}_{0\tau}'|$ (b) and $|\mathbf{H}_{0\tau}| = |\mathbf{H}_{0\tau}'|$ (c).

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